Question 2.

Step 1: DEFINE YOUR PROPOSITIONS.

Let p be "the prime minister of Canada supports military intervention in Syria."
q be "the president of the US supports military intervention in Syria."
r be "the president of the Russian Federation supports military intervention in Syria."

Step 2: REPRESENT ARGUMENT SYMBOLICALLY.

$$
\begin{array}{ll}
p \veebar q & \text{Premise} \\
q \to \neg r & \text{Premise} \\
\underline{\neg q} & \underline{\text{Premise}} \\
\therefore\ p \wedge r & \text{Conclusion.}
\end{array}
$$

Step 3: CONSTRUCT TRUTH TABLE.

				Premise	Premise	Premise	Conclusion.
p	q	r	$\neg r$	$p \veebar q$	$q \to \neg r$	$\neg q$	$p \wedge r$
T	T	T	F	F	F	F	T
T	T	F	T	F	T	F	F
T	F	T	F	T	T	T	T
T	F	F	T	T	T	T	F
F	T	T	F	T	F	F	F
F	T	F	T	T	T	F	F
F	F	T	F	F	T	T	F
F	F	F	T	F	T	T	F

Step 4: CONCLUSION.

Recall, an argument is valid if the conclusion is true whenever the premises are true.

Since we have a case where the premises are all true but the conclusion is not $\therefore$ the argument is not valid/invalid/fallacy.

Question 4

Modus Tollens.

Show that modus tollens is a valid argument.

$$[(p \to q) \wedge (\neg q)] \to (\neg p) \Leftrightarrow \mathbb{T}.$$

Proof:

$$\begin{aligned}
\left[(p \to q) \wedge (\neg q)\right] \to (\neg p) &\Leftrightarrow \left[(\neg p \vee q) \wedge (\neg q)\right] \to \neg p && \text{Imp. Rule.} \\
&\Leftrightarrow \left[(\neg q) \wedge (\neg p \vee q)\right] \to \neg p && \text{Comm.} \\
&\Leftrightarrow \left[(\neg q \wedge \neg p) \vee \underbrace{(\neg q \wedge q)}\right] \to \neg p && \text{Distrib.} \\
&\Leftrightarrow \left[(\neg q \wedge \neg p) \vee \mathbb{F}\right] \to \neg p && \text{Contradiction.} \\
&\Leftrightarrow \left[\neg q \wedge \neg p\right] \to \neg p && \text{Identity} \\
&\Leftrightarrow \neg(\neg q \wedge \neg p) \vee \neg p && \text{Imp.} \\
&\Leftrightarrow (\underbrace{\neg\neg q} \vee \underbrace{\neg\neg p}) \vee \neg p && \text{De Morgan} \\
&\Leftrightarrow (q \vee p) \vee \neg p && \text{Double Neg (x 2)} \\
&\Leftrightarrow q \vee \underbrace{(p \vee \neg p)} && \text{Assoc.} \\
&\Leftrightarrow q \vee \mathbb{T} && \text{Excluded Middle} \\
&\Leftrightarrow \mathbb{T} && \text{domination.}
\end{aligned}$$

♦

Another proof can be found in Kohar's Basic Discrete Mathematics, p. 47-48 (on reserve at the library).